Question 1

$P = -0.001x^2 - 0.2x + 8$

Market price: $ 18 per disc

Consumer Surplus:

$$\int_0^{q^*} d(q)\,dq - p^*q^*$$

$$18 = -0.001x^2 - 0.2x + 8$$

$$-0.001x^2 - 0.2x - 10 = 0$$

Solving Quadratically: $x = -100$

$$P(18) = -0.001(18)^2 - 0.2(18) + 8$$

$$= 4.076$$

$$\int_0^{18} (-0.001x^2 - 0.2x + 8)dx - 18 \times 4.076$$

$$= \left[\frac{-0.001}{3}x^3 - \frac{0.2}{2}x^2 + 8x\right]_0^{18}$$

$$= (-1.944 - 32.4 + 144) - 73.368$$

$$= \$\ 36.288$$

Question 2

$$P = 600e^{-0.04x}$$

(a) $500 = 600e^{-0.04x}$

$$5/6 = e^{-0.04x}$$

$$\ln(5/6) = -0.04x$$

$x = 4.558$ units of hundreds

$= 456$ units

(b) Selling price set at $500 per laptop.

$$\int_0^{4.558} 600e^{-0.04x}\,dx - 4.558 \times 500$$

$$= 600\int_0^{4.558} e^{-0.04x}\,dx \implies u = -0.04x,\ dx = \frac{du}{-0.04}$$

$$= -15000\int_0^{4.558 \times -0.04} e^{u}\,du = -15000\left[e^{-0.04x}\right]_0^{4.558 \times -0.04} - 4.558 \times 500$$

$$= 2499.981 - 2279$$

$$= \$220.981$$

Question 3

$P = \sqrt{36 + 1.8x}$

$$\text{Producer surplus} = p^* q^* - \int_0^{q^*} S(q)\, dq$$

$q = 36 + 1.8x$

$x = 15$ units

$$P_S = 15 \times q - \int_0^{15} \sqrt{36 + 1.8x}$$

Take $\int_0^{15} \sqrt{36 + 1.8x}$

$u = 36 + 1.8x$

$\frac{du}{dx} = 1.8$; $dx = \frac{du}{1.8}$

$36 + (1.8 \times 15) = 63$

$$= \frac{1}{1.8} \int_{36}^{63} \sqrt{u}$$

$$= 5/9 \left[\frac{2}{3} u^{3/2} \right]_{36}^{63}$$

$$= \frac{5}{9} \left[\frac{2}{3} (36 + 1.8x)^{3/2} \right]_{36}^{63}$$

$= 676.335 - 374.823$

$= 105.203$

$= 135 - 105.203$

$= \$29.8$

Question 4.

$$P = 100 + 80e^{0.05x}$$

(a) $400 = 100 + 80e^{0.05x}$

$300 = 80e^{0.05x}$

$3.75 = e^{0.05x}$

$\ln(3.75) = 0.05x$; $x = 26.435$ units

$= 26.435 \times 100 =$ 2644 computers

(b) $400 \times 26.435 - \int_0^{26.435} 100 + 80e^{0.05x}$

Take $\int_0^{26.435} 100 + 80e^{0.05x}$

$= \int_0^{26.435} 100\,dx + 80\int_0^{26.435} e^{0.05x}\,dx$

$= \left[100x\right]_0^{26.435} + 80 \times 20\left[e^{0.05}\right]_0^{1.3217}$

$= 2643.5 + 4399.9648 = 7043.452$

$= 400 \times 26.435 - 7043.452$

$= \$3530.55$

Question 5

$R(t) = 100,000$

$IR = \%$ $= 0.04$

$t = 5$

$$Fv = e^{rM} \int_0^M R(t)\, e^{-rt}\, dt$$

$$= e^{0.04 \times 4} \int_0^4 100,000\, e^{-0.04t}\, dt$$

$$= e^{0.16}\, 100,000 \int_0^4 e^{-0.04t}\, dt$$

Take $\int_0^4 e^{-0.04t}\, dt$ $\quad u = -0.04t$ $\quad \frac{du}{dt} = -0.04$, $dt = \frac{du}{-0.04}$

$$= \int_0^{-0.16} e^{u} \frac{du}{-0.04} = -25 \int_0^{-0.16} e^{u}\, du$$

$$= -25 \left[e^{-0.04t} \right]_0^{-0.16}$$

$$= -25(-1.006 + 1) = +0.161$$

$25(1 - e^{-4/25}) = 3.6964$

$= 1.173510 \times 100,000 \times 3.6964$

$= \$433775.16$

$= \$433775$

Question 6

$R(t) = 150000$

$r = 7\% = 0.07$

$t = 4$

$$PV = \int_0^4 (t+1)\, e^{-0.07}\, dt$$

$$PV = \int_0^4 150000\, e^{-0.07t}\, dt$$

$$= 150000 \int_0^4 e^{-0.07t}\, dt$$

$u = -0.07t$

$\frac{du}{dt} = -0.07$

$dt = \frac{du}{-0.07}$

$$= \frac{-150000}{0.07} \int_0^{-0.28} e^{u}\, du$$

$$= \frac{-150000}{0.07} \left[e^{-0.07t} \right]_{-0.28}^{0}$$

$$= \frac{-150000}{0.07} \left[1 - e^{0.0196} \right]$$

$$= 2142857.1428 \times 0.24422$$

$$= \$\, 523328.57$$